Thermal/Structural Dynamic Analysis via a Transform-Method-Based Finite-Element Approach

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The paper describes a generalized transform-method-based finite-element methodology for interfacing interdisciplinary areas, with emphasis on thermal/structural dynamic applications. The purpose of the paper is to present an alternate approach for interdisciplinary analysis via a common numerical methodology for each interdisciplinary area, and therein outline the potential of the proposed formulations. Characteristic features of the methodology and fundamental concepts are highlighted for the interdisciplinary areas of heat transfer and structural dynamics. Details of a Laplace-transform-based finite-element interface methodology are described and applied to discrete-type flexural configurations subjected to rapid surface heating. Results obtained demonstrate excellent agreement in comparison with analytic solutions and/or conventional finite-element formulations. The methodology offers potential for thermostructural applications, and the concepts outlined, it is hoped, will provide avenues to interface other interdisciplinary areas as well.

Nomenclature

A = cross-sectional area
$[B_s]$ = strain-displacement function matrix
$[C_s]$ = damping matrix of structural element
$[D]$ = elasticity matrix
e = element
E = modulus of elasticity
$\{F_T\}$ = equivalent nodal thermal force vector
h = depth of structural member
I = moment of inertia
$[K_s]$ = structure stiffness matrix
$[\bar{K}_s]$ = transformed structure stiffness matrix
$[\bar{K}]$ = transformed thermal stiffness matrix
ℓ = length of structural member
m = mass per unit length
$[M_s]$ = mass matrix
$[\bar{N}]$ = transformed thermal or structural interpolation function matrix
$\{\bar{q}\}$ = transformed vector of nodal displacements and rotations
$\{\bar{Q}\}$ = transformed heat load vector
s = Laplace transform variable in transformed domain
$\{T\}$ = temperature vector
t = time
$\{u\}$ = vector of nodal displacements
w = lateral deflection of structural member
x, y, z = Cartesian coordinates
α, α_c = thermal diffusivity, thermal expansion coefficient
β = $(ms^2/4EI)^{1/4}$, see Eq. (15)
$\{\varepsilon\}$ = strain vector
θ = slope
λ = $(s/\alpha)^{1/2}$, see Eq. (12)

Subscripts

s = structure
T = temperature

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Introduction

THERMAL/STRUCTURAL modeling and analysis are of considerable practical importance to structural designers concerned with problems related to temperature-induced displacements and stresses and the associated structural dynamic response due to thermal considerations. It has been known for quite some time that when long, slender structural configurations are subjected to the rapid surface heating typical in several large space structures in orbit, thermally induced oscillations take place and may become significant. The problem of structural vibration due to thermal effects was originally introduced by Boley[1] in 1956. His studies showed that structural inertia effects in most analyses can be disregarded, thereby permitting a quasi-static thermal/structural analysis, except in uncommon cases of slender structures such as very thin beams and plates under rapid surface heating. Although complex aerospace structures have potential for thermally induced oscillations, as noted by several researchers,[2,3] there is little systematic fundamental research, particularly on thermal/structural approaches for both static and structural dynamic responses for complex structural configurations. A literature review[3–5] cites the importance of efficiently and effectively predicting the static and dynamic characteristics of large flexible structures and complex structural configurations because routine experimental and/or ground tests are highly impractical and difficult to simulate. This is especially the case when the structure's dynamic responses are due solely to thermal effects. A typical example is that of a microwave radiometer spacecraft,[3] as shown in Fig. 1. The spacecraft system is representative of complex structures characterized by lattice structures, pretensioned cables, flexural beam members, and tubular tension and compression members. The complexity of typical structural configurations influenced by the interdisciplinary nature of thermal/structural mechanics significantly influences the response characteristics and makes the analysis of such problems challenging.

The present paper describes a generalized transform-method-based finite-element methodology[6] for interfacing interdisciplinary areas, with emphasis on thermal/structural dynamic applications. The purpose of this paper is to present an alternate methodology for interdisciplinary thermal/structural dynamic analysis and therein outline the advantages of the proposed transform-method-based finite-element approach for combined thermostructural analysis. The research is aimed directly at providing avenues and concepts via a common numerical methodology for the automated solution of each of the transient heat-transfer and structural dynamic responses.

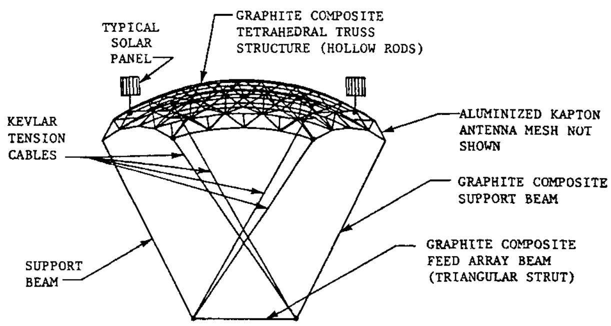

Fig. 1 Microwave radiometer spacecraft, Ref. 3.

Highlights and details of the approach for complex thermal/structural dynamic problems are described through applications to realistic and practical problems. Comparisons with analytic solutions and/or conventional finite-element schemes are made whenever possible. The potential of the approach for other interdisciplinary areas is also discussed.

Generalized Transform-Method-Based Finite-Element Interface Methodology

The computational aspects involving fluid/thermal/structural interactions are currently open to limited solution techniques owing to lack of appropriate methodologies to date. The present paper concerns interfacing interdisciplinary areas via transform methods and finite-element techniques. The approach presented is very general and can be effectively used for fluid/thermal, fluid/structure and thermal/structural problems. In view of the complexities involved in interfacing interdisciplinary areas, the introduction of an effective generalized methodology, henceforth referred as the "transform-method-based finite-element (TMFE) interface methodology," for combined analysis is presented and is a viable alternative to existing techniques. A unique feature of the approach is the use of a common numerical methodology for each of the interdisciplinary areas via transform methods in conjunction with finite-element formulations. In the TMFE technique, Laplace, Fourier, or any other appropriate transforms can be used in conjunction with finite elements for combined interdisciplinary analysis. Of the several transform methods that can be used in conjunction with the TMFE methodology, the Laplace-transform finite-element (LTFE) interface methodology and the Fourier-transform finite-element (FTFE) interface methodology are typical classifications of finite-element formulations in the corresponding transformed domains, respectively, the only difference being the numerical transform inversion technique used to obtain the response of the combined analyses. The finite-element formulations in the transformed domain are referred to as "transfinite formulations," and the corresponding finite elements are referred to as "transfinite elements." In fact, the approach may be used in conjunction with other numerical methods such as finite-difference methods and boundary element methods. The remainder of this paper concerns the interdisciplinary areas of heat transfer and structural mechanics, using the TMFE interface methodology.

The conventional thermal/structural finite-element formulations (in fact, any numerical formulations) for studying the transient thermal/structural response of structural configurations are quite complex and very time-consuming for several reasons, principally the time-dependent nature of the problem. This is especially true when analyzing large complex structural configurations due to thermal effects, which require analyses to be carried out for long durations, thereby escalating computational (CPU) times and analysis costs. Furthermore, these analyses require step-by-step time-marching algorithms, estimation of time steps, etc. And, in certain cases, if proper care is not taken in the associated modeling and formulations, the analyses may lead to oscillatory solution behavior. In view of these conditions, the TMFE interface methodology for efficiently interfacing the heat-transfer and structural disciplines for combined thermal/structural analysis using both thermal finite-element and structural finite-element formulations is presented herein.

Solution Sequence: Thermal Analysis/Structural Analysis

The solution sequence presented herein can be used for any one of the following: 1) thermal analysis 2) structural analysis or 3) unified thermal and structural analysis. The steps in the solution sequence follow:

1) Divide the physical domain of interest into finite elements appropriate to the problem under consideration.

2) Select any appropriate transform method and apply this with respect to time to the governing differential equations and boundary conditions for the given problem; obtain the transformed equations.

3) Select interpolation functions, which are functions of only the space variables for the dependent variable variation, by either solving for the general solution of the transformed differential equation or assuming a suitable function to approximate the dependent variable variation in the transform domain.

4) Apply the method of weighted residuals (MWR) or a variational approach to derive the necessary finite-element matrices and evaluate them, using the interpolation functions obtained in step 3.

5) Sum the contributions of all elements in the transformed domain, and form the system equations.

6) Apply boundary conditions, and solve the resulting system of finite-element equations for the unknowns in the transformed domain.

7) Obtain the solution in the time domain by numerically inverting the solution obtained in step 6. For unified thermal/structural analysis, the transformed thermal loads are consistently evaluated and directly transferred to the structural model in the transform domain itself without the need of intermediate intra-analysis numerical inversion.

The selection of interpolation functions or basis functions is an important consideration in accurately predicting the dependent variable variation. It should be emphasized that this selection in the transform domain via the proposed formulations is arbitrary as in conventional finite-element formulations. Although not necessary, for certain classes of problems and structural configurations, the selection of the interpolation functions via the general solution of the transformed differential equation is advantageous in that the corresponding interpolation functions generally yield more accurate results for the same element degree of freedom. For example, for discrete-type structural configurations modeled by one-dimensional thermal or structural elements (see examples presented), such a selection not only reduces significantly the total number of system degrees of freedom or system model size but also achieves improved accuracy in the response. This is because these elements better represent the dependent variable variation within each element. And, as will be shown in the examples to be presented, the solution response obtained via a refined mesh of conventional formulations approaches the response predicted by the models composed of elements formulated via the general solution of the transformed equations. It should be noted, however, that the authors are not advocating the use of such basis functions in general via the proposed approach but are merely stating an advantage for such discrete-type structural configurations. In general, the use of polynomial functions is entirely permissible in the transform domain via the proposed TMFE approach. If the selection of the interpolation functions via the general solution of the transformed equation is not feasible, in general, any appropriate interpolation functions may be used to approximate the dependent variable variation within an element as in conventional formulations. Although, for such a selection, the corresponding discretization needs to be refined to achieve similar accuracy, the general advantages of the TMFE approach outlined in this paper are still retained. Once the type of elements and their interpolation functions have been selected, the associated element matrices derived via the MWR or a variational approach can be evaluated and then assembled, following the usual procedures of assembly.

After developing the finite-element formulations in the transform domain, the system of equations is then solved in the transform domain itself. Therein, the solution response is obtained by employing an inverse numerical transform. This is an important step in the solution methodology. For demonstrating the basic concepts in this paper, the Laplace-transform-based finite-element methodology[6] mentioned earlier is illustrated. Numerical inversion of transform methods has been in existence for quite some time; it was originally used in applied mathematics and later applied to modal response studies and related problems.[7-10] In Ref. 11, various techniques of the numerical Laplace inversion process are described in detail. Currently, several inversion techniques are available, each having its pros and cons in terms of accuracy and efficiency. An excellent reference on this topic is Davies and Martin.[11] For the examples presented in this paper, the numerical inversion algorithm of Durbin[12] was employed because of the excellent accuracy of the method. Durbin's fast Laplace inversion technique uses both finite Fourier sine and cosine transforms to yield increased accuracy and efficiency. This inversion technique is an improvement of Dubner and Abate's approach.[13] We have found Durbin's inversion technique[12] to give very accurate results consistent with observations cited in Refs. 12 and 13. The major advantages of this approach are: 1) the error bound on the inverse of a function is independent of time instead of being exponential in time, and 2) the corresponding trigonometric series obtained for the function in the transform domain is valid for the entire period of the series. The accuracy of the inversion algorithm increases with the total number of terms used in the series. The disadvantages we noticed include: 1) the need to use more terms for increased accuracy and 2) that the algorithm seemed to predict less accurate results at the very last time point of the duration. (However, when we increased the last time point slightly, the results for the previous last time point were as accurate as those during the total transient.) For obtaining the real time response of a transformed function at desired times of interest, the corresponding summation series formulated for the function is multiplied by a factor that is directly proportional to the exponential of the desired time of interest and inversely proportional to the total duration of the transient. The inversion technique is relatively straightforward to implement, and readers interested in the inversion procedure are directed to Ref. 12 for further details. Although the present study focuses on linear thermal/structural dynamic analysis, research is currently under way for applicability to nonlinear problems.

In the next section, details of the Laplace-transform-based finite-element (LTFE) formulations applicable to thermal, structural, and unified thermal/structural dynamic analysis are presented.

Thermal Analysis: LTFE Formulation

As indicated earlier, the principal advantage in the use of the Laplace transform (or any other appropriate transform) method to transient thermal analysis is that the time parameter t can be eliminated, thereby requiring the finite-element formulation of a steady-type problem. Equations (1) are the fundamental formulas commonly used in conjunction with the Laplace-transform method.

$$\bar{T}(s) = L\{T(t)\} = \int_0^\infty e^{-st}T(t)\,\mathrm{d}t \tag{1a}$$

$$L\{\dot{T}(t)\} = s\bar{T}(s) - T(t)|_{t=0} \tag{1b}$$

where $T(t)$ is any real function of t and $\bar{T}(s)$ is the Laplace transform of $T(t)$. After dividing the physical domain into finite elements appropriate for a given problem, the first step for the thermal analysis involves applying the Laplace transform in time to the governing heat-transfer equations. As mentioned in the earlier section, the selection of the element interpolation functions or shape functions, which are functions of space variables, is an important step for accurately predicting the temperature distribution. This selection may be achieved by either solving the transformed differential equation or assuming an appropriate function to approximate the temperature distribution. If the interpolation functions are obtained from the solution of the differential equations in the transformed domain, then an exact element, and hence an exact solution, results (the solution is exact within the framework of the inversion process). Either the method of weighted residuals or a variational approach can be applied to derive the appropriate element matrices.

With the elimination of the variable t, the steady-type solution for various values of s will yield the unknown temperature T, employing the inverse Laplace transform. It is now possible to obtain the nodal temperatures at independent values of t and, unlike conventional finite-element formulations, the computation of temperatures T does not need the calculation of temperatures at previous times. That is, the solution response at desired times of interest during any given transient can be readily obtained. Furthermore, the flexibility of restarting at a suitable intermediate time using the recent solution vector as initial conditions is also feasible as in conventional formulations. Through an appropriate inversion technique, the solution response can now be readily obtained. The accuracy of the solution obtained depends on the numerical inversion process.[11]

Structural Analysis

For an uncoupled thermal/structural dynamic analysis, the displacement response $u(t)$ is computed from the general structural dynamics equations

$$[M_s]\{\ddot{u}\} + [C_s]\{\dot{u}\} + [K_s]\{u\} = \{F_T(t)\} \tag{2}$$

where $[M_s]$ is the mass matrix, $[C_s]$ the damping matrix, and $[K_s]$ the structure stiffness matrix. Only the equivalent nodal force vector $\{F_T(t)\}$, due to time-dependent member temperature distributions, is shown above. In general, the mass matrix is independent of temperature, but the damping and stiffness matrices are implicit functions of temperature because of the temperature dependence of material properties. The numerical solution of Eqs. (2) requires periodic updating of the damping and stiffness matrices to account for temperature variations. When the dynamic effects in Eqs. (2) are neglected, the resulting quasi-static response yields

$$[K_s]\{u\} = \{F_T(t)\} \tag{3}$$

which requires a sequence of static analyses to be performed at selected times during the thermal transient. The solution of Eqs. (3) is less time-consuming and inexpensive than the solution of Eqs. (2) and is a reasonable approximation, provided thermally induced oscillations are not significant. The equivalent nodal force vector $\{F_T(t)\}$ is given by

$$\{F_T(t)\} = \int_v [B_s]^T[D]\{\varepsilon_t - \varepsilon_0\}\,\mathrm{d}v \tag{4}$$

where $\{\varepsilon_t - \varepsilon_0\}$ is the total change in the thermal strain vector from an initial strain, $[D]$ the elasticity matrix, and $[B_s]$ the strain-displacement function matrix.

Structural Analysis: LTFE Formulation

As in the thermal analysis, the introduction of the Laplace transform technique to general structural dynamic equations has the primary advantage in that the time parameter t can be eliminated, thereby requiring the solution of a static-like problem in the transform domain. The general procedure involves application of the Laplace transform in time to the governing equations of motion. The resulting transformed governing equation can now be solved to obtain the necessary interpolation functions, or an appropriate displacement function can be assumed to approximate the displacement variation within an element. Through the application of the method of weighted residuals (MWR) or a variational approach, the resulting finite-element matrices for structural analysis can now be easily obtained in the transformed domain. After the problem is solved in the transformed domain, the solution is then numerically inverted to obtain the transient structural response.

For effectively interfacing the heat-transfer and structural disciplines in a combined thermal/structural analysis by the LTFE interface methodology, it is not necessary to obtain the temperature distribution in the time domain through numerical inversion (unless one wishes to know the thermal response) since the corresponding structural problem is also formulated and solved in the transformed domain. To study the structural dynamic response due to thermal effects, it is necessary to invert numerically only in the final structural formulation.

Beam Element Formulation

To illustrate the basic concepts of the LTFE interface methodology for unified transient thermal/structural analysis, the finite-element formulations representing a beam element are developed herein. For demonstration purposes, the formulations presented utilize interpolation functions derived via the general solution of the transformed governing equations, although similar formulations could be developed using other types of interpolation functions. The major advantages of the transform-method-based finite-element methodology are nevertheless retained. A unique feature of the element formulated herein is the use of an exact temperature variation to compute the "thermal beam" finite-element matrices and the exact displacement variation to compute the "structural beam" finite-element matrices to predict the structural response due to thermal effects.

Thermal Formulation

Consider a uniform flexural beam member of length ℓ, cross-sectional area A, and height h, subjected to a transient surface heating as shown in Fig. 2. The governing heat-transfer equations are given by

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5}$$

where $T = T(x, y, t)$ and α is the thermal diffusivity. Applying the Laplace transform with respect to time for zero initial conditions yields

$$\frac{\partial^2 \bar{T}}{\partial x^2}+\frac{\partial^2 \bar{T}}{\partial y^2}=\left(\frac{s}{\alpha}\right)\bar{T} \tag{6}$$

where

$$\bar{T}(x, y, s) = \int_0^\infty T(x, y, t)e^{-st}\,\mathrm{d}t$$

At this juncture, the finite-element formulations are introduced. The element temperature variation $\bar{T}_e$ in the transformed domain is expressed as

$$\bar{T}_e = [\bar{N}]\{\bar{T}\}_e \tag{7}$$

where $[\bar{N}]$ are typical element interpolation functions and $\{\bar{T}\}_e$ the nodal temperatures. Following the MWR or the variational approach, the finite-element matrices for Eq. (6) are obtained in the form

$$[\bar{K}]_e\{\bar{T}\}_e = \{\bar{Q}\}_e \tag{8}$$

where $[\bar{K}]_e$ is the appropriate element stiffness matrix and $\{\bar{Q}\}_e$ is the heat load vector. For simplicity in illustrating the basic concepts of the LTFE methodology, the flexural member is assumed to be subjected to a constant surface heating suddenly applied and uniformly distributed along its length, as shown in Fig. 3. Thus, Eq. (6) reduces to

$$\frac{\partial^2 \bar{T}}{\partial y^2}-\left(\frac{s}{\alpha}\right)\bar{T}=0 \tag{9}$$

where

$$\bar{T}(y, s) = \int_0^\infty T(y, t)e^{-st}\,\mathrm{d}t$$

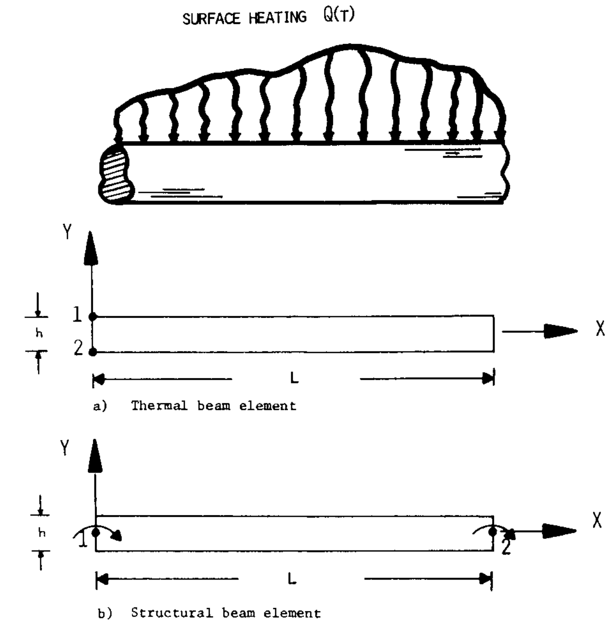

Fig. 2 Flexural member subjected to heating and typical thermal and structural beam elements: a) thermal beam element; b) structural beam element.

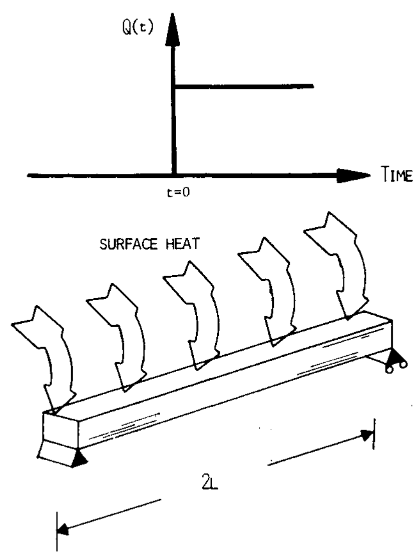

Fig. 3 Simply supported structural member with constant surface heating.

The general solution to Eq. (9) is

$$\bar{T}(y,s) = C_1 e^{\lambda y} + C_2 e^{-\lambda y} \tag{10}$$

Using the above general solution, we have the element temperature variation derived as

$$\bar{T} = [\bar{N}]\{\bar{T}\}_e = [\bar{N}_1 \bar{N}_2]\begin{Bmatrix}\bar{T}_1\\ \bar{T}_2\end{Bmatrix} \tag{11}$$

where the interpolation functions are given by

$$\bar{N}_1 = \frac{\sinh\lambda(h-y)}{\sinh(\lambda h)} \tag{12a}$$

$$\bar{N}_2 = \frac{\sinh(\lambda y)}{\sinh(\lambda h)} \tag{12b}$$

and

$$\lambda = (s/\alpha)^{1/2} \tag{12c}$$

Following the MWR, the element matrices are of the form

$$[\bar{K}]_e\{\bar{T}\}_e = \{\bar{Q}\}_e \tag{13}$$

where $[\bar{K}]_e$ has the contribution of conduction stiffness and convective-like stiffness matrices and $\{\bar{Q}\}_e$ is the corresponding heat load vector. The system equations are next obtained following the usual assembly procedures. At this juncture, after developing the finite-element formulations in the transform domain, the system of equations are then solved in the transform domain itself, and the temperatures so obtained are directly used for evaluating the thermal load vector in the structural analysis without the need for inversion during this transfer.

Structural Formulation

The governing equation of flexural motion without damping for a beam member of length ℓ, mass per unit length m, and subjected to surface heating is given by

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} = 0 \tag{14}$$

where $w = w(x, t)$ is the lateral deflection.

For zero initial conditions, applying the Laplace transform with respect to time yields

$$\frac{\partial^4 \bar{w}}{\partial x^4} + 4\beta^4 \bar{w} = 0 \tag{15}$$

where

$$\bar{w}(x,s) = \int_0^\infty w(x,t)e^{-st}\,\mathrm{d}t$$

and

$$\beta^4 = ms^2/4EI$$

The general solution can be expressed as

$$\bar{w}(x,s) = [e^{\beta x}\cos\beta x \;\; e^{\beta x}\sin\beta x \;\; e^{-\beta x}\cos\beta x \;\; e^{-\beta x}\sin\beta x] \times \begin{Bmatrix}C_1\\ C_2\\ C_3\\ C_4\end{Bmatrix} = [\bar{g}]\{c\} \tag{16}$$

Applying local element conditions

$$\bar{w} = \bar{w}_1,\ \bar{\theta} = \bar{\theta}_1, \text{ at } x = 0 \tag{17a}$$

$$\bar{w} = \bar{w}_2,\ \bar{\theta} = \bar{\theta}_2, \text{ at } x = \ell \tag{17b}$$

we have

$$\bar{w}(x,s) = [\bar{N}]_e\{\bar{u}\}_e \tag{18}$$

where $[\bar{N}]_e$ are element interpolation functions and $\{\bar{u}\}_e$ the element nodal displacements and rotations. Following the MWR, the structural element matrices are represented by

$$[\bar{K}_s]_e\{\bar{u}\}_e\{\bar{F}_T\}_e \tag{19}$$

where $[\bar{K}_s]_e$ is the element structural stiffness matrix and $\{\bar{F}_T\}_e$ the element load vector due to thermal effects. The structural stiffness matrix has been developed in Ref. 7. If the element thermal axial force effects are neglected, only temperature-induced moments are present and are given as

$$\bar{M}_T = \int_A E\alpha_c \bar{T} y\,\mathrm{d}A \tag{20}$$

where $\bar{T}(y,s)$ is transferred from the thermal finite-element formulation advocated earlier. Axial effects may be considered through a more general structural dynamic stiffness matrix as shown in Ref. 14. The system equations are next obtained following the usual procedures of assembly. To study the structural dynamic response due to thermal effects, it is now necessary to invert numerically only the final structural formulation.

Applications

In this section, several test cases are presented to demonstrate the basic concepts of the proposed formulations and evaluate the transient thermal and structural dynamic response. To sharpen the focus of the present study and demonstrate the methodology, discrete-type flexural configurations subjected to transient thermal loading are analyzed. These configurations are modeled via the finite-element formulations presented in the preceding section (although similar formulations could be developed with different approximating functions, as discussed in an earlier section) to outline clearly the advantages of achieving reduced model sizes and superior accuracy for discrete-type flexural configurations in contrast to larger models and refined conventional schemes. The test cases include: 1) a long, slender, simply supported flexural member subjected to a sudden ramp heating, 2) a cantilever with/without an end mass subjected to a ramp heating, 3) transient thermal response of a repetitive lattice structure, and 4) thermal/structural dynamic response of a segment of a wing structure. In all the examples presented, comparisons are made whenever possible with available solutions and/or conventional finite-element formulations.

Simply Supported Beam

A simply supported beam (Fig. 4) of length 2ℓ, mass per unit length m, bending rigidity EI, rectangular cross-sectional area A, and depth h was subjected to a sudden constant surface heat input on the top surface; the bottom surface was assumed to be insulated. The data assumed for the computations are shown in Fig. 4. The beam was modeled using two elements across the span so that the deflection at the middle could be evaluated. Figure 4 shows the comparative solutions for the transient structural dynamic response. Since the beam used exact interpolation functions for the thermal and structural elements, respectively, this leads to a relatively smaller model size, and the results are also in excellent agreement.

Cantilever With/Without End Mass

A flexural cantilever member of length ℓ, mass per unit length m, bending rigidity EI, rectangular cross-sectional area A, and depth h was subjected to a sudden ramp heating on the top surface; the bottom surface was assumed insulated. Two cases of the cantilever were analyzed (Fig. 5). The first was

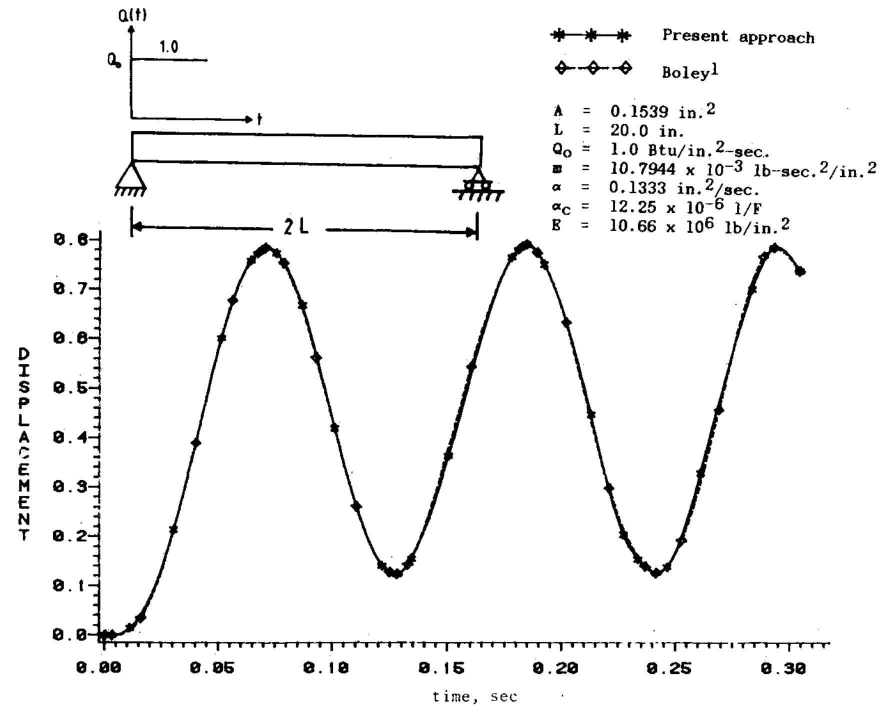

Fig. 4 Comparative solutions for transient thermal/structural dynamic response of simply supported flexural member.

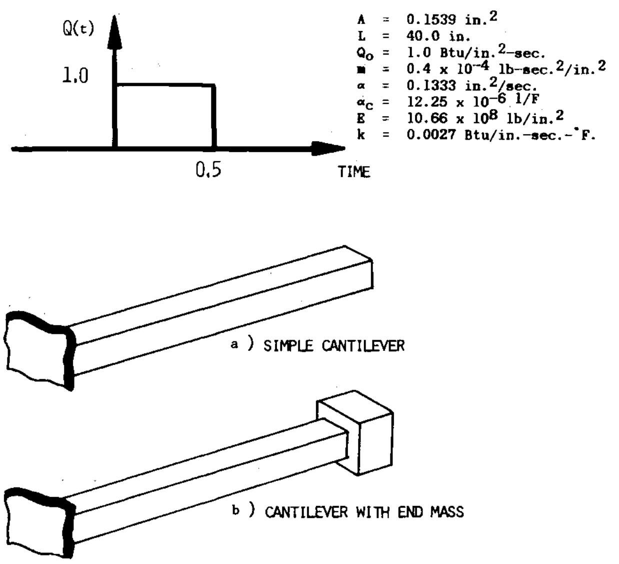

Fig. 5 Cantilever structural members subjected to sudden ramp heating.

modeled without any end mass, and the second case with an end mass. For both cases, only one beam element (derived in the previous section) was used in each of the models, respectively. Since the results for a simply supported flexural member demonstrated excellent agreement, no comparisons were made for these examples. Figure 6 shows the temperature variation through the thickness of the member for the cases when heating was applied ($t = 0.3$ s) and when there was no heating ($t = 0.42$ s). In Figs. 7 and 8, the transient structural response of the tip is shown for each of the two cases. It is interesting to note that the responses shown in each of the two cases are not only out of phase with each other but, at longer times during the transient, the deflection of the member with an end mass is greater than the member without an end mass due to inertia effects. The superimposed structural dynamic response is shown in Fig. 9.

Transient Thermal Response of Repetitive Lattice Structure

The top surface members of a three-dimensional repetitive lattice structure are subjected to a sudden constant surface heat input as shown in Fig. 10. The tetrahedral-type repetitive configuration is formed by structural members lying along the intersections of repeating tetrahedrons connected at their bases and their vertices. The structure was modeled by: 1) one element per member (formulated via the general solution of the transformed equation), and 2) ten conventional linear thermal elements (NASTRAN) per member in order to capture the detailed temperature distributions. Figure 10 shows the comparative thermal response for the temperature history for a typical node (node 12), together with the results of a refined conventional finite-element model (NASTRAN) that used one element and ten thermal elements per member, respectively. As shown in Fig. 10, the results of the more refined conventional model approach the response predicted by the formulations derived earlier in which, for this type of structural configuration, the selection of the interpolation functions via the general solution of the transformed equation not only significantly reduces the model size but also achieves excellent accuracy.

Thermal/Structural Dynamic Response of a Segment of a Wing Structure

A segment of a wing structure (Fig. 11), fixed at one end and free at the other, was subjected to a sudden constant surface heat input on the top surface; the bottom surface was assumed insulated. The data assumed for the computations are also shown in Fig. 11. The wing structure was first modeled by elements formulated via the general solution of the transformed equation and therein used one element per member, resulting in a small model size. For comparison purposes, the configuration was also modeled by five elements per member with NASTRAN, which uses conventional beam element formulations. The vertical deflection at the free end (node 9) was evaluated in order to assess the transient thermal/structural dynamic response of the model. Figure 12 shows the comparative response. The results for the five-elements-per-member NASTRAN model (Fig. 12) and the present formulations are in excellent agreement. It should be noted that as the number of elements per member in the NASTRAN model were in-

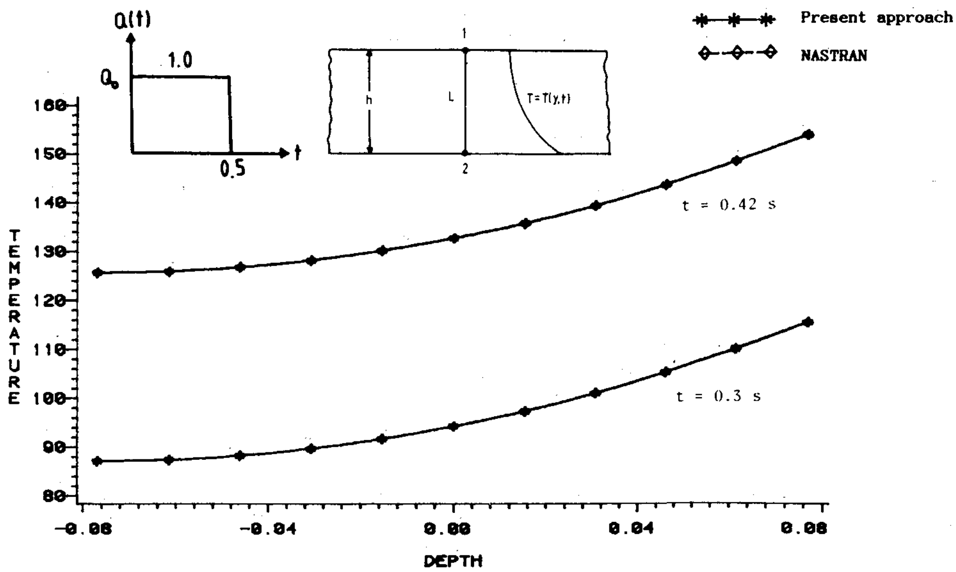

Fig. 6 Comparative transient thermal response through depth at $t = 0.3$ and 0.42 s.

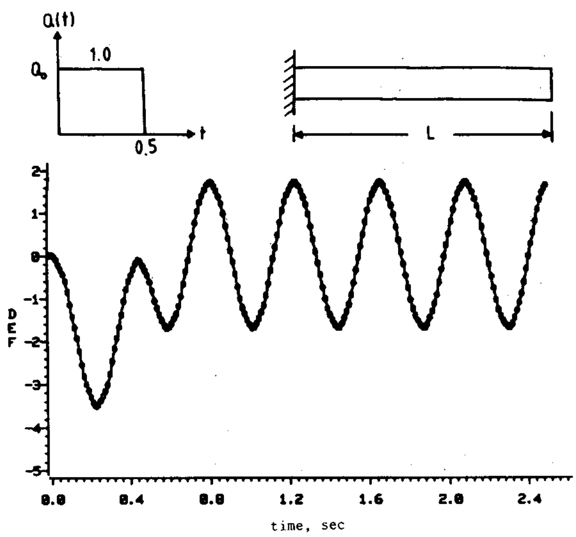

Fig. 7 Thermal/structural dynamic response of cantilever subjected to sudden ramp heating.

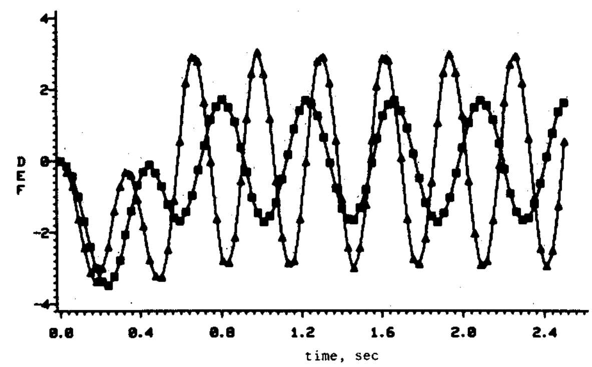

Fig. 9 Superimposed thermal/structural dynamic response of cantilever with/without end mass.

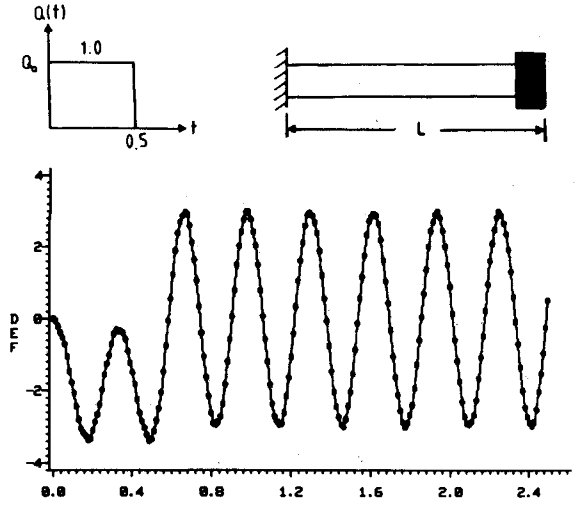

Fig. 8 Thermal/structural dynamic response of cantilever with end mass subjected to sudden ramp heating.

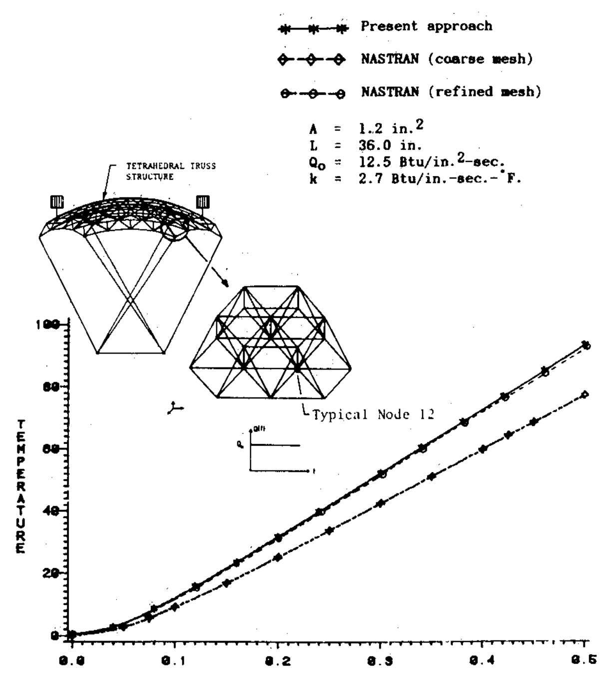

Fig. 10 Comparative transient thermal response of repetitive lattice structure (node 12).

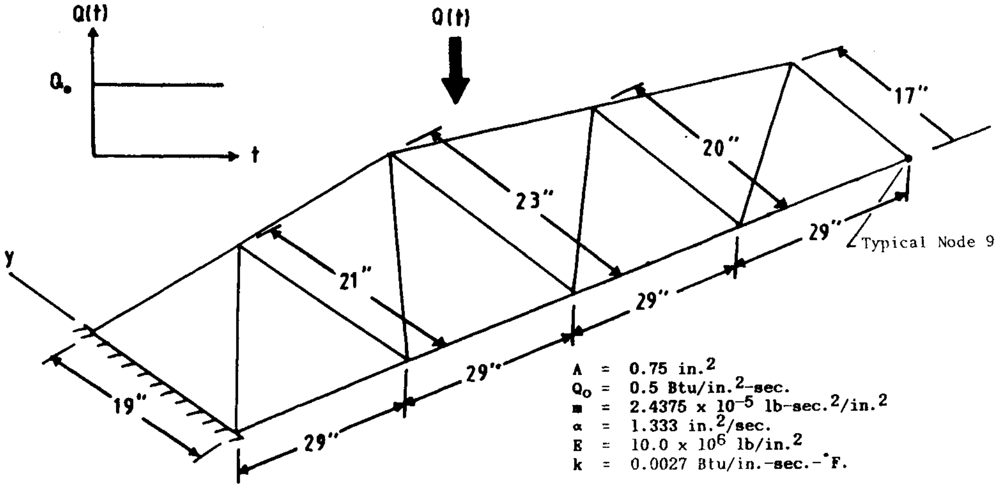

Fig. 11 Wing structure with suddenly applied surface heating load.

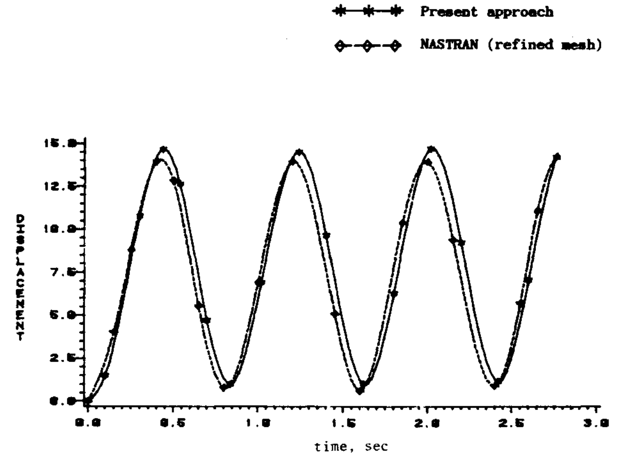

Fig. 12 Comparative thermal/structural dynamic response for wing structure (node 9).

creased, the results (not shown) were found to approach the present solution, which required fewer degrees of freedom than the much larger and detailed NASTRAN model to achieve similar accuracy.

Concluding Remarks

The paper presented an alternative and viable methodology using transform-method-based finite-element formulations for interfacing the interdisciplinary areas of heat transfer and structural dynamics. A unique feature of the approach is the use of a common numerical methodology for each of the individual disciplines of thermal/structural dynamics via transform methods in conjunction with finite-element formulations. Characteristic features of the transform-method-based finite-element (TMFE) methodology are highlighted, and fundamental concepts and details of the solution scheme are outlined. The application of a Laplace-transform-based finite-element interface methodology for discrete-type flexural configurations subjected to rapid heating demonstrated excellent agreement of results with available solutions and/or with conventional finite-element formulations. The approach presented provides features and alternate concepts for thermal/structural dynamic models and offers significant potential for extensions to other interdisciplinary areas as well.

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